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Nonlinear vibration isolator with adjustable restoring force

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Abstract

This paper presents a vertical quasi-zero stiffness (QZS) vibration isolator with a mechanism for adjusting restoring force. QZS vibration isolators have high initial stiffness and QZS around the static equilibrium position. This way, excessive deformation due to self-weight can be avoided while having enough vibration reduction capability to dynamic excitations. One of the main issues left for QZS vibration isolators is the difficulty in keeping the vibration reduction capability when the vibration isolated object is replaced. In such a case, adjustment of its restoring force becomes necessary in accordance with the self-weight of the newly placed vibration isolated object. This paper attempts to address this issue by proposing a mechanism that enables quick and easy adjustment of the restoring force of a QZS vibration isolator. The proposed mechanism consists of cranks and a screw jack. With the present

mechanism, the restoring force provided by horizontally placed springs can be converted into the vertical restoring force of the vibration isolator. In the conversion, the vertical resisting force can be adjusted simply by applying and removing torque to the screw jack to change and hold the angle of inclined bars placed in the cranks. In this study, a prototype of a class of QZS vibration isolator having the proposed mechanism is produced. Shaking table tests are performed to demonstrate the efficacy of the present mechanism, where the produced prototype is subjected to various sinusoidal and earthquake ground motions. It is demonstrated through the shaking table tests that the produced prototype can reduce the response acceleration within the same tolerance even when the mass of the vibration isolated object is changed.

Keywords: nonlinear vibration isolator; vertical vibration isolator; constant-force spring; adjustable restoring force

1. Introduction

Passive vibration isolation has been an active area of research, and a variety of passive vibration isolators have been developed so far. For horizontal vibration isolation, low stiffness elements such as springs are installed between the base of a vibration isolator

and the objects to be isolated. The low stiffness elements prevent vibration energy from being transmitted into the isolated object. On the other hand, if low stiffness elements are used for vertical vibration isolation, large deformations take place due to the self-weight of the isolated objects. This makes vertical vibration isolation much more difficult than horizontal vibration isolation.

To overcome the difficulty, quasi-zero-stiffness (QZS) vibration isolators have been studied for many years [1-25]. QZS vibration isolators are designed so that their restoring forces satisfy the following two conditions: (1) initial stiffness, which resists self-weight, is large, and (2) the tangent stiffness, which resists dynamic load, is close to zero around the static equilibrium position. This way, transmission of vibration energy due to dynamic load can be prevented while excessive deformation due to self-weight can be avoided. In most QZS vibration isolators, such nonlinear restoring forces are realized by effective use of geometric nonlinearity [1-22]. In the past studies, magnets [23,24] or shape memory alloys [25] were also used to obtain such nonlinear restoring forces.

One of the main issues left for the QZS vibration isolators is the difficulty in adjusting their restoring forces to the change of the self-weight, or the mass, of the isolated object. When the mass changes, it is necessary for the QZS vibration isolators

to adjust their restoring force by, e.g., replacing springs. The use of balance weight is another way of dealing with the mass change. In either case, it is not easy and takes considerable time for the adjustment of the nonlinear restoring force of a QZS vibration isolator. To the authors' knowledge, however, no studies have addressed this issue.

This study presents a QZS vibration isolator with a mechanism that enables easy and quick adjustment of its nonlinear restoring force. In this paper, first, a mechanism is proposed to change the nonlinear restoring force. Also presented is a combination of the proposed mechanism and a vertical vibration isolator with flag-shaped restoring force, which belongs to a class of QZS vibration isolators and was proposed in the authors' previous work [26,27]. To show the efficacy of the present mechanism, a prototype of the proposed vibration isolator is produced to perform quasi-static and dynamic loading tests. Numerical simulations are also conducted to compare the test and simulation results with theoretical predictions.

2. Mechanisms

2.1. Mechanism for adjusting restoring force

Figure 1(a) illustrates a schematic view of the present vibration isolator. As shown in Figure 1(b), the proposed isolator is divided into three portions. The upper portion is a

table on which the isolated objects are placed. The upper portion consists of one horizontal top plate and two vertical hexagonal plates. The middle portion is composed of two shafts connected by elements to provide a nonlinear restoring force. Constant-force springs, coil springs, and wires are examples of such elements. Here, constant-force springs are composed of rolled spring steel and have constant restoring force regardless of their stretch [26,27]. At both ends of each shaft, inner and outer roller bearings are installed. In the lower portion, one screw jack, two linear bushes, and four cranks are attached to the base plate. Here, the screw jack and the linear bushes are connected by two beams. The beams are connected to the cranks using pin connections. Half of the length of the screw jack is threaded in the opposite direction to that of the other half. The distance between the two beams can be changed by rotating the screw jack while keeping the geometry of the mechanism symmetric. The positions of the beams are automatically locked simply by removing the torque applied to the screw jack. This way, the angle of the inclined bar in each crank can be changed and held simply by applying and removing the torque to the screw jack. As depicted in Figure 1(a), the inclined sides of the hexagonal plates are placed on the inner bearings of the shafts, and the outer bearings of the shafts are placed on the inclined bars of the cranks. With such a composition, the horizontal restoring force provided by the middle portion can be

converted into the vertical restoring force. In the conversion, furthermore, the vertical resisting force can be adjusted simply by changing the angle of the inclined bars of the cranks, which is the key ingredient of the present vibration isolator.

The relationship between the restoring force f of the vibration isolator and the restoring force n provided by the middle portion shown in Figure 2 is derived below.

Here, the positive direction of f is compression while that of n is tension. As shown in Figures 2(b) and 2(c), let the angles of bar AB and the inclined side of the upper table be θ and γ , respectively, with respect to the vertical. Figure 3 illustrates the internal forces acting on the right shaft shown in Figure 2(b). Figure 3(a) illustrates the internal force acting on each inner bearing from the inclined side of the upper table. Figure 3(b) shows the internal force acting on each outer bearing from inclined bar AB of the crank. As shown in Figure 3(c), the force-balance equation in the horizontal direction for the shaft can be written as

$$n = \frac{1}{2} \left(\frac{1}{\tan \theta} + \frac{1}{\tan \gamma} \right) f. \quad (1)$$

Assume that the mass of the shaft is negligibly small. Then, Eq. (1) is valid even under dynamic condition. Thus, the amplification factor α is defined as

$$\alpha(\theta) = \frac{f}{n} = \frac{2}{1/\tan \theta + 1/\tan \gamma}. \quad (2)$$

Figure 2(c) illustrates the schematic view of a crank. Bars AB and AE are connected by

a pin at point A. Bars CD and AB are also connected by a pin at point D. Bar AE is connected rigidly to the base plate at point E. The lengths of AD, AE, and CD are r , s , and l , respectively. Let h indicate the distance between point C and the base plate. Let the distance between point C and bar AE be d , which is the key variable to adjust θ . Then, the geometric relationship between θ and d is

$$l^2 = (d + r \sin \theta)^2 + (s - h - r \cos \theta)^2. \quad (3)$$

Suppose that d is given. Then, θ can be calculated by solving Eq. (3), and α can be obtained by substituting the calculated θ into Eq. (2).

2.2. Mechanism for flag-shaped restoring force

Figure 4 shows a mechanism for providing a flag-shaped restoring force [26,27]. The relative vertical displacement between the upper table and the base is defined as u , whose positive direction is contraction. The relative displacement between the two shafts is defined as w , whose positive direction is extension. Figure 4(a) depicts the position where the upper table is at the upper limit of the stroke of the vibration isolator. At this position, both u and w are set as $u=w=0$. Figures 4(b) illustrates the static equilibrium position when an object is placed on the upper table, where $u=u_{se}$ and $w=w_{se}$. Here, m denotes the mass of the object and g indicates the gravitational

acceleration. Figure 4(c) illustrates the position where the upper table reaches the lower limit of the stroke, where $u=u_{max}$ and $w=w_{max}$.

In the following, the variation of n is considered when w is increased from 0 to w_{max} . When $w=0$, the restoring force n changes from 0 to N_1 , where N_1 is the sum of the capacities of constant-force springs 1a and 1b. At all the positions when $0 < w < w_{max}$, both constant-force springs 1a and 1b provide restoring forces equal to their capacities. On the other hand, constant-force spring 2, which is connected to the shafts via a coil spring and a wire in series, provides a restoring force between 0 and N_2 , where N_2 denotes the capacity of constant-force spring 2. When w is small, no restoring force is provided by constant-force spring 2 due to the sag of the wire. In this case, n is expressed as $n=N_1$. When the upper tables moves downward beyond the position where the wire sag stops to take place, the length of the coil spring changes linearly with respect to w . The present vibration isolator is designed so that the static equilibrium position lies in this linear range. Here, the coil spring is used to avoid high frequency accelerations caused by sudden, or discontinuous, change of the restoring force. When the upper table moves downward further and exceeds the position where the restoring force of the coil spring reaches N_2 , the restoring force provided by constant-force spring 2 becomes N_2 . In this case, n is expressed as $n= N_1+ N_2$.

Figures 5(a) and 5(b) illustrate the $f-u$ and $n-w$ relationships. These figures also illustrate the shifts of axes. In this paper, $\Delta(\bullet)$ indicates that the reference position of the variable is the static equilibrium position. More specifically, Δf , Δn , Δu , and Δw are defined as

$$\Delta f = f - mg, \quad \Delta n = n - mg / \alpha, \quad \Delta u = u - u_{se}, \quad \Delta w = w - w_{se}. \quad (4)$$

Such shifts of axes are introduced here to consider the vibration around the static equilibrium position. Figure 5(c) depicts the geometric relationship between u and w when the upper table is moved downward while the base is fixed. The thicker line indicates the upper side of bar AB, and the thinner lines indicate the inclined side of the upper table before and after the movement. From this figure, the geometric relationship can be written as

$$u = \frac{1}{2} \left(\frac{1}{\tan \gamma} + \frac{1}{\tan \theta} \right) w = \frac{1}{\alpha} w. \quad (5)$$

Figure 6(a) shows the relationship between the shifted restoring force Δn_{NF} and the shifted relative displacement Δw , where no friction is assumed. Let k be the stiffness of the coil spring. Then, the relationship can be written as

$$\Delta n_{NF} = n_{NF} - \frac{mg}{\alpha} = \begin{cases} N_1 - mg / \alpha & \text{if } -w_{se} < \Delta w < w_L \\ k \Delta w & \text{if } \Delta w_L \leq \Delta w \leq \Delta w_U \\ N_1 + N_2 - mg / \alpha & \text{if } w_U < \Delta w < w_{\max} - w_{se} \end{cases}, \quad (6)$$

where

$$\Delta w_U = (N_1 + N_2 - mg / \alpha) / k, \quad \Delta w_L = (N_1 - mg / \alpha) / k. \quad (7)$$

Figure 6(b) illustrates the relationship between the friction force q and $\Delta \dot{w}$ assuming Coulomb's friction [28]. Here, the overdot indicates the differentiation with respect to time. The friction force q can be written as

$$q = \begin{cases} -Q & \text{if } \Delta \dot{w} < 0 \\ 0 & \text{if } \Delta \dot{w} = 0 \\ Q & \text{if } \Delta \dot{w} > 0 \end{cases}. \quad (8)$$

Then the shifted restoring force Δn including Coulomb's friction can be written as

$$\Delta n(\Delta w, \Delta \dot{w}) = \Delta n_{NF}(\Delta w) + q(\Delta \dot{w}), \quad (9)$$

Figure 7 illustrates this relationship. The restoring force curve shown in Figure 7(c) is called as “flag-shaped restoring force” because the shape of the restoring force curve in the region of $\Delta n > 0$ and $\Delta w > 0$ looks like a flag [29].

From Eq. (2), the relationship between Δn and Δf can be written as

$$\Delta f = \alpha \Delta n. \quad (10)$$

And, from Eq. (5), the relationship between Δu and Δw can be written as

$$\Delta u = \frac{1}{\alpha} \Delta w. \quad (11)$$

From Eqs. (10) and (11), Δf can be written as

$$\Delta f(\Delta u, \Delta \dot{u}) = \begin{cases} \alpha(N_1 + q(\alpha \Delta \dot{u})) - mg & \text{if } -u_{se} < \Delta u < w_L / \alpha \\ \alpha^2 k \Delta u + \alpha q(\alpha \Delta \dot{u}) & \text{if } w_L / \alpha \leq \Delta u \leq w_U / \alpha \\ \alpha(N_1 + N_2 + q(\alpha \Delta \dot{u})) - mg & \text{if } w_U / \alpha < \Delta u < u_{\max} - u_{se} \end{cases}, \quad (12)$$

Note that, when mg changes, α should be adjusted so that the following inequality

$$\alpha(N_1 + Q) < mg < \alpha(N_1 + N_2 - Q), \text{ or } \frac{mg}{N_1 + N_2 - Q} < \alpha < \frac{mg}{N_1 + Q} \quad (13)$$

holds to make the vibration isolator self-centering, or to make the upper table return to the static equilibrium position when the external force other than the gravity force is removed.

The strengths of the present mechanism and the flag-shaped restoring force can be summarized as follows:

1. The use of constant-force springs allows the mechanism to have a long stroke of more than several centimeters while keeping the mechanism compact.
2. Existence of plateau in the restoring force curve limits the response acceleration within a specified tolerance regardless of the amplitude and frequency of an input motion.
3. The flag-shaped restoring force provides the isolator with the self-centering capability. This is because static equilibrium can be achieved only at the origin of the restoring force curve. The self-centering capability prevents the displacement from accumulating in one direction in vibration, which may be seen when friction damping is applied [29].
4. The flag-shaped restoring force avoids resonance. The proposed isolator responses

linearly in the neighborhood of the static equilibrium point. In this linear range, the isolator has a potential of resonance. Nevertheless, this does not lead to serious problems because the displacement amplitude of interest in this paper is well beyond the linear range. Even if resonance takes place in the linear range, the equivalent natural frequency (computed either by the secant stiffness or by the tangent stiffness) decreases significantly as the displacement amplitude increases beyond the linear range. The change of the equivalent natural frequency with respect to the displacement amplitude avoids resonance.

2.3. Upper bound of response acceleration

As pointed out in Section 2.2, one of the key features of the present vibration isolator is that it can limit the response acceleration within a specified tolerance regardless of the magnitude and frequency of an input motion. This subsection presents basic equations for this feature. Suppose that the vibration isolator is subject to the (absolute) ground acceleration \ddot{u}_g . Then, the equation of motion for the present vibration isolator can be written as

$$m(\Delta \ddot{u} + \ddot{u}_g) + c\Delta \dot{u} + \Delta f(\Delta u, \Delta \dot{u}) = 0. \quad (14)$$

Assume that the vibration isolator is designed so that c is negligibly small. Then, from

220 Eqs.(8), (12), and (14), the magnitude of the absolute response acceleration $|\Delta\ddot{u} + \ddot{u}_g|$

221 can be bounded as

$$222 \quad |\Delta\ddot{u} + \ddot{u}_g| \leq h(\alpha), \quad (15)$$

223 where

$$224 \quad h(\alpha) = \max \left(g - \frac{\alpha}{m}(N_1 - Q), \frac{\alpha}{m}(N_1 + N_2 + Q) - g \right). \quad (16)$$

225 From Eqs. (15) and (16), it can be said that the proposed vibration isolator can specify
 226 the peak response accelerations by adjusting α/m . This implies that the present
 227 vibration isolator can reduce the peak response acceleration to the same value by
 228 adjusting the amplification factor α when m is changed.

229 Consider the range of variation of $h(\alpha)$ when α changes in the range of Eq. (13).

230 Both $g - \alpha(N_1 - Q)/m$ and $\alpha(N_1 + N_2 + Q)/m - g$ are linear functions of α , and
 231 hence $h(\alpha)$ is a piecewise linear function of α . The range of variation of $h(\alpha)$ can be
 232 obtained as

$$233 \quad \frac{N_2/2 + Q}{N_1 + N_2/2} g \leq h(\alpha) < \frac{N_2}{N_1 + Q} g. \quad (17)$$

234 It should be noted here that, from Eqs. (15) to (17), the response acceleration can be
 235 reduced to less than $N_2 g / (N_1 + Q)$ regardless of m as far as the assumptions made in
 236 this section are valid.

237

3. Experiments

3.1 Prototype design

Figure 8 shows a photograph of a prototype of the present vibration isolator produced for testing the effectiveness of the proposed mechanisms. The height of the vibration isolator at the static equilibrium position is about 400 mm. To suppress the rocking response of the upper table, 2 plates are fixed with pin connections to each side of the vibration isolator. Note that, although the screw jack is rotated by hand in the present prototype of the vibration isolator, it is possible to use a combination of sensing devices and an electric motor to rotate the screw jack for sensing the mass change and adjusting the restoring force automatically.

The parameters for the mechanism for the flag-shaped restoring force are as follows. The nominal values of N_1 and N_2 are 235.5 N and 31.4 N, respectively, and that of k is 1.5 N/mm. It is difficult to determine the value of Q prior to the quasi-static loading tests because friction takes place at many locations. Hence, as explained later in Section 3.2, the reference value of Q is determined as 6.8 N from the quasi-static loading tests. Substituting these values into Eq. (17), the theoretical prediction of the upper bound of the response acceleration can be obtained as 0.13 g.

The parameters for the mechanism for changing the restoring force are provided

256 as $\tan \gamma = 1.6$, $l = 150$ mm, $r = 75$ mm, and $s - h = 150$ mm. Figure 9 shows the
 257 relationship between the amplification factor α and d . Here, recall that d is the
 258 distance between point C and bar AE as shown in Figure 2. Table 1 shows the reference
 259 values that define the amplified restoring forces, where $N_{1(\alpha)}$, $N_{2(\alpha)}$, $Q_{(\alpha)}$, $u_{R(\alpha)}$, and
 260 $k_{(\alpha)}$ are defined as

$$261 \quad N_{1(\alpha)} = \alpha N_1, \quad N_{2(\alpha)} = \alpha N_2, \quad Q_{(\alpha)} = \alpha Q, \quad u_{R(\alpha)} = \frac{w_U - w_L}{\alpha}, \quad k_{(\alpha)} = \alpha^2 k. \quad (18)$$

263 **3.2 Quasi-static loading tests**

264 To obtain the parameters for constructing a numerical model of the proposed vibration
 265 isolator, quasi-static loading tests are carried out. The quasi-static loading tests are
 266 performed by changing the value of d parametrically from 15 mm to 65 mm with an
 267 increment of 5 mm. In each quasi-static loading test, forced displacement is applied to
 268 the upper table manually. The origin is set to the equilibrium position for each test. The
 269 displacement of the upper table is obtained by using non-contact laser displacement
 270 sensors. The restoring force f is obtained by the load cells inserted between the ground
 271 and the base of the vibration isolator.

272 Figure 10(a) shows the restoring force curves obtained from the quasi-static
 273 loading tests for the cases of $d = 60$, 40, and 20 mm. Here, the results are shown only

for the region of $-50\text{mm} < \Delta u < 50\text{mm}$ while the applied displacement amplitude of Δu was more than 50mm. From the figure, it is clear that the restoring force is amplified by adjusting the value of d . Figure 10(b) shows the numerical models of the restoring force curves, where $S_{1(\alpha)}$ and $S_{2(\alpha)}$ are the upper and lower values of the restoring force when $\Delta u \leq -u_{R(\alpha)}/2$, and $S_{3(\alpha)}$ and $S_{4(\alpha)}$ are the values when $u_{R(\alpha)}/2 \leq \Delta u$. Table 2 summarizes the values of the parameters that determine the flag-shaped restoring force. The procedure used for determining these values is summarized as follows:

1. The values of $S_{1(\alpha)}$ and $S_{2(\alpha)}$ are determined by averaging the upper and lower values of the restoring force curve, respectively, in the range of $-50\text{ mm} \leq \Delta u \leq -u_{R(\alpha)}/2$. The values of $S_{3(\alpha)}$ and $S_{4(\alpha)}$ are determined similarly in the range of $u_{R(\alpha)}/2 \leq \Delta u \leq 50\text{ mm}$.

2. The amplified capacities $N_{1(\alpha)}$ and $N_{2(\alpha)}$ are obtained as

$$N_{1(\alpha)} = \frac{S_{1(\alpha)} + S_{2(\alpha)}}{2}, \quad N_{2(\alpha)} = \frac{S_{3(\alpha)} + S_{4(\alpha)}}{2} - N_{1(\alpha)}. \quad (19)$$

3. The amplified stiffness $k_{(\alpha)}$ around the static equilibrium position is determined by

$$k_{(\alpha)} = \frac{N_{2(\alpha)}}{u_{R(\alpha)}}. \quad (20)$$

4. The amplified friction force $Q_{(\alpha)}$ is determined by

$$Q_{(\alpha)} = \frac{(S_{1(\alpha)} - S_{2(\alpha)}) + (S_{3(\alpha)} - S_{4(\alpha)})}{2}. \quad (21)$$

The experimentally observed restoring force curves shown in Figure 10(a) are not so smooth as their numerical models shown in Figure 10(b) because it is difficult to realize such ideal restoring force curves shown in Figure 9(b) due to many factors. Uncertainty of the restoring force provided by the constant-force springs is one of the main reasons. Backlash and friction in the bearings and pin-connections are other obstacles to realize such ideal restoring force curves.

Figure 11 compares the nominal values of $N_{1(\alpha)}$ and $N_{2(\alpha)}$ with those obtained experimentally. Good agreement can be seen between these values. In Figure 12, the cross mark plots the relationship between $N_{1(\alpha)}$ and $Q_{(\alpha)}$, both of which are obtained from the experiments. The solid line shows the linear relationship

$$Q_{(\alpha)} = \beta N_{1(\alpha)}, \quad (22)$$

where the value of β is obtained by the least-squares fit as 0.029. The reference value $Q_{(\alpha)}$ shown in Table 1 is obtained by multiplying the obtained value of β and the nominal value of $N_{1(\alpha)}$.

3.3. Shaking table tests

Figure 13 illustrates the configuration of the experimental setup. Only vertical ground motions are applied to the vibration isolator. To investigate the vibration reduction capability of the present vibration isolator when the mass of the isolated object is changed, shaking table tests are performed for the cases of $d = 60, 40, \text{ and } 20 \text{ mm}$. These values correspond to the amplification factor α of 1.25, 1.57, and 1.91, and the mass m of 32.0 kg, 40.2 kg, and 49.0 kg, respectively. Acceleration sensors are installed on the upper table of the vibration isolator to measure response acceleration (RA), and on the shaking table to record ground acceleration (GA). Non-contact laser displacement sensors are fixed to a measurement frame to measure response absolute displacement (RAD) and ground displacement (GD). Response relative displacement (RRD) is obtained by subtracting GD from RAD. The primary natural frequencies in the neighborhood of the static equilibrium position are 2.74, 3.26, and 3.68 Hz for the cases of $d = 60, 40, \text{ and } 20 \text{ mm}$, respectively. These values are obtained from the free vibration measurements.

Table 3 summarizes input excitations. The input waves are 2, 3, 4, 5, and 6 Hz sine waves, and the vertical, or up-down (UD), component of the earthquake ground motion recorded at K-NET Ojiya station in Japan during the 2004 Mid Niigata Prefecture earthquake. The peak ground accelerations (PGAs) are normalized to 0.25 g

and 0.50 g for 2 Hz sine wave, 0.25 g, 0.50 g, 0.75 g, and 1.00 g for 3, 4, 5, and 6 Hz sine waves. The PGAs are normalized to 0.50 g, 0.75 g, and 1.00 g for the K-NET Ojiya UD earthquake ground motion, whose PGA is 0.83 g.

Table 4 reports all the PGAs and the peak response accelerations (PRAs) measured in the shaking table tests. Due to the limitations of the capability of the shaking table, there are small differences between the intended and the measured PGAs. Figure 14 compares the PRAs with the PGAs. As can be observed in Table 4 and Figure 14, the PRAs are reduced to under 0.2 g for all the sine waves except the case of $d = 20$ mm and S4-4 input. On the other hand, the PRAs were dropped to under 0.15 g for all the scaled earthquake ground motions. These results demonstrate that the proposed vibration isolator can reduce the response acceleration effectively in a wide range of input frequency and amplitude. Table 5 summarizes the peak response absolute displacements (PRADs) and the peak ground displacements (PGDs). It is seen in Table 5 that significant reduction was achieved in PRAD to all sine waves. On the other hand, reductions are not observed for the K-Net Ojiya UD records. In fact, the PRADs were amplified by the vibration isolator. Figure 15 shows examples of the time histories of the GA, RA, GD, RAD, RRD for the case of $d = 40$ mm. The input excitations are S4-4 and O-3. Figure 15 shows that the RAs were reduced sufficiently during the entire

excitation for both cases. The reduction of the RAD to S4-4 excitation and the amplification to O-3 excitation were confirmed through the plots of the time histories.

4. Numerical Simulations

In this section, numerical simulations are performed to compare their results with the experimental results. For the numerical simulations, the values of $N_{1(\alpha)}$, $N_{2(\alpha)}$, $Q_{(\alpha)}$, and $k_{(\alpha)}$ obtained from the quasi-static loading tests (shown in Table 2) are used. The values of the mass used in the shaking table test are substituted into m , i.e., 32.04 kg, 40.24 kg, and 48.95 kg for the cases of $d = 60$ mm, 40 mm, and 20 mm, respectively. The damping factor c is assumed to be 0.01 Ns/mm for $d = 60$ mm, 0.04 Ns/mm for $d = 40$ mm, and 0.06 Ns/mm for $d = 20$ mm. These values are determined by trial and error to fit the simulations results to the experimental data. To avoid singular points in numerical simulations, the friction force q is approximated by q^* , defined as

$$q^* = \begin{cases} -Q & \text{if } \Delta\dot{w} < -Q/b \\ b\Delta\dot{w} & \text{if } -Q/b \leq \Delta\dot{w} \leq Q/b, \\ Q & \text{if } \Delta\dot{w} > Q/b \end{cases} \quad (23)$$

The slope b is determined as 1 Ns/mm for all the cases by trial and error as well. Numerical integrations are performed using the Runge-Kutta method.

Figure 16 compares the time history responses of the RAs calculated from the

simulations with the experimental results when the input excitations are S4-4 and O-3 and the value of d is 40 mm. Figure 16 demonstrates that the RA waves obtained from the simulations agree reasonably well with the experimental results.

The PRAs obtained from the experiments and the simulations for the cases of $d = 60, 40,$ and 20 mm are compared in Figure 17, where the input excitations are S2-2, S3-4, S4-4, S5-4, S6-4, and O-3. For reference, the theoretical prediction of the upper bound of the response acceleration obtained by using Eq. (17) is plotted by the solid line. As shown in Figure 17, the theoretical prediction agrees well with the simulation results. On the other hand, there are a couple of cases where the PRAs obtained from the shaking table tests exceed the theoretical predictions.

Figure 18 compares the response acceleration spectra [30] to study the potential of the measured waves to shake the objects placed on the shaking table and the upper table of the isolator. In the figure, the case of S4-4 and $d = 20$ mm where the highest PRA are measured is investigated and the damping ratio is assumed as 5 %. As can be seen from Figure 18, the difference between the experimental and simulation results of the response acceleration appears mainly in the frequency range higher than 20 Hz. This verifies that the low frequency vibrations, which are important for avoiding large displacement of the isolated object, are suppressed effectively by the present vibration

isolator, although scrutiny of the source of the unintended high-frequency vibrations is still necessary. Backlash and friction in the bearings and pin-connections are potential sources of the unintended high-frequency vibrations.

5. Conclusions

This study has presented a passive vertical vibration isolator with nonlinear restoring force adjustable to the change of the mass of the isolated object. A simple mechanism to change the nonlinear restoring force has been introduced. One of the main features of the present vibration isolator is that quick and easy adjustment is possible when the mass of the isolated object is changed while keeping the peak response acceleration within the same tolerance. The efficacy of the proposed vibration isolator has been demonstrated through shaking table tests and numerical simulations, where the vibration isolator was subjected to sine waves and scaled earthquake ground motions under different values of the mass of the isolated object.

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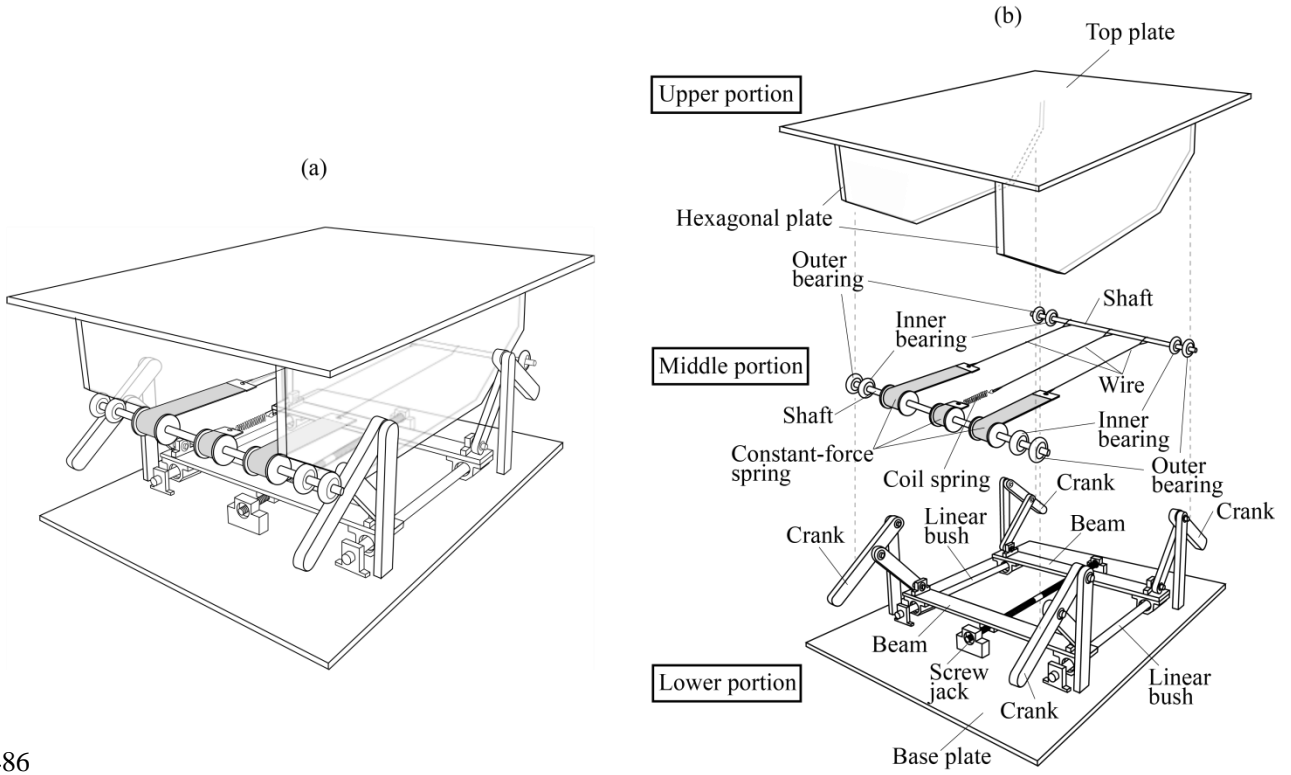


Figure 1: Schematic view of the proposed vibration isolator. (a) Whole view. (b) Each component.

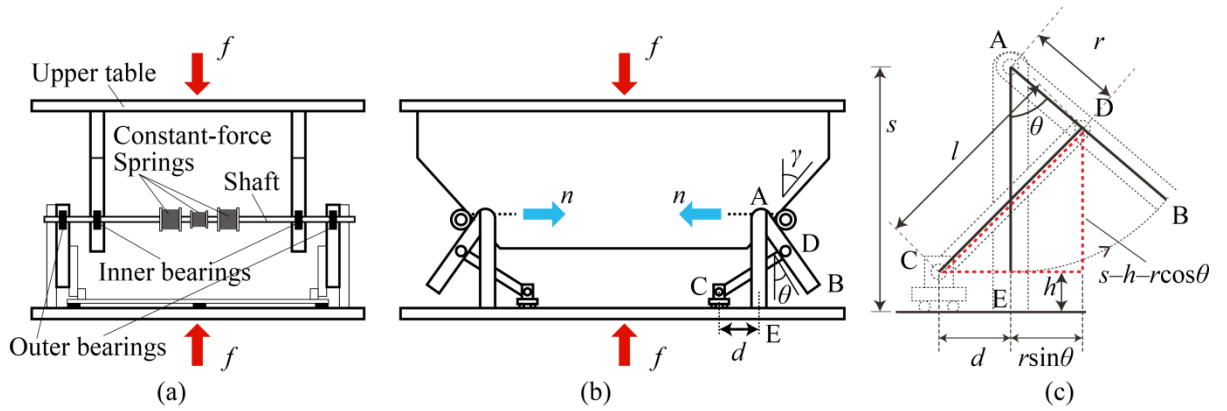


Figure 2: Schematic illustration of the proposed device. (a) Front side. (b) Lateral side. (c) Crank.

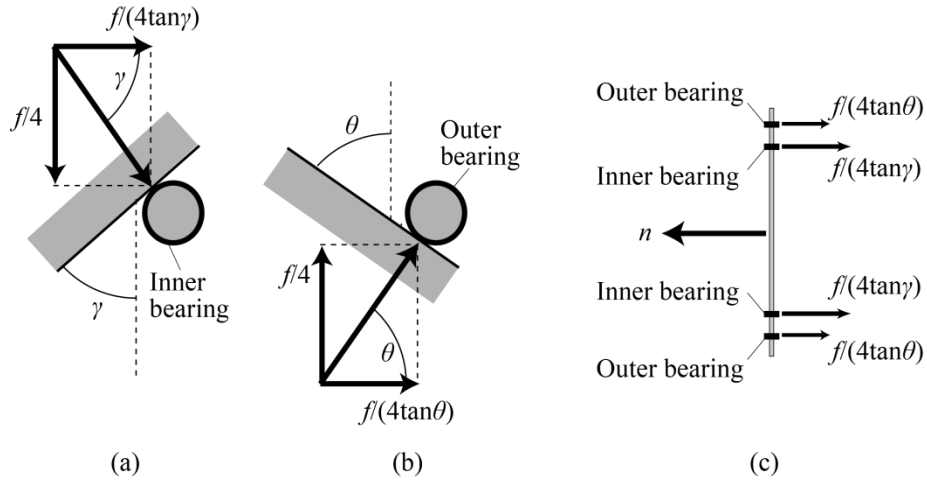


Figure 3: Internal force on the right shaft. (a) Inner bearing. (b) Outer bearing. (c) Equilibrium of horizontal forces acting on the right shaft.

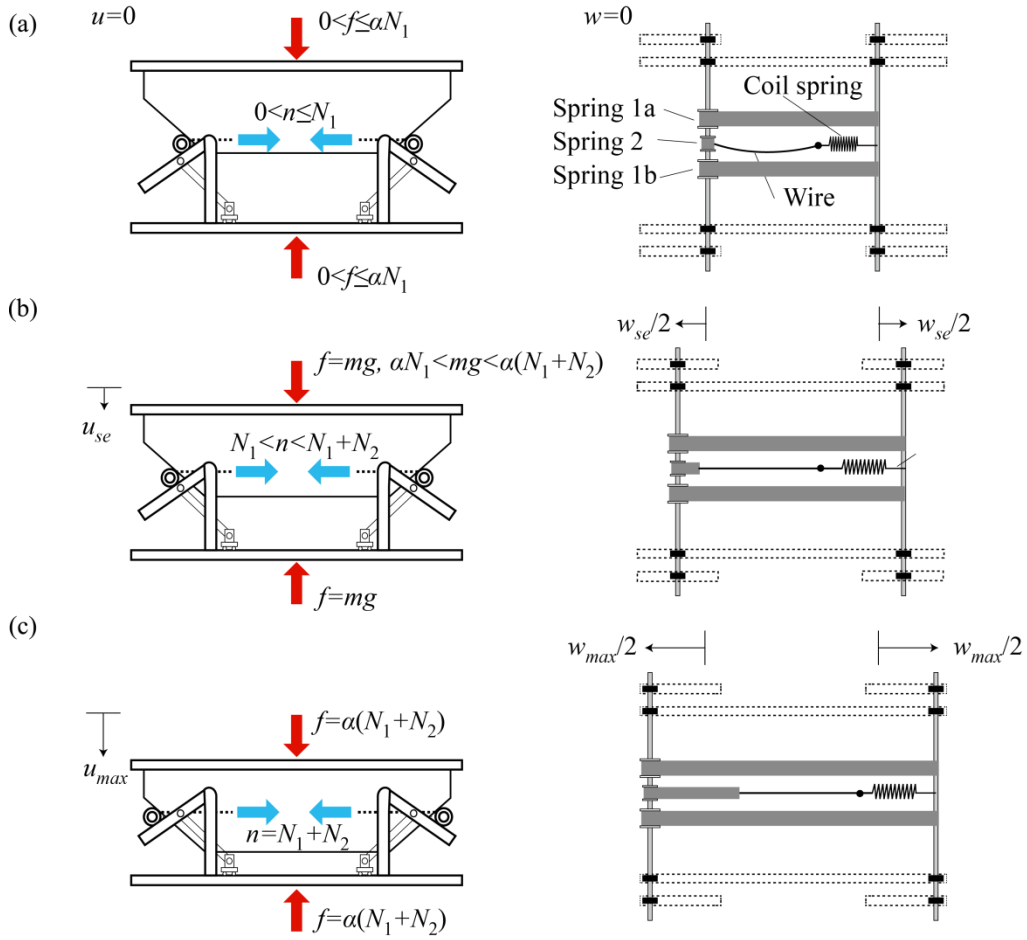
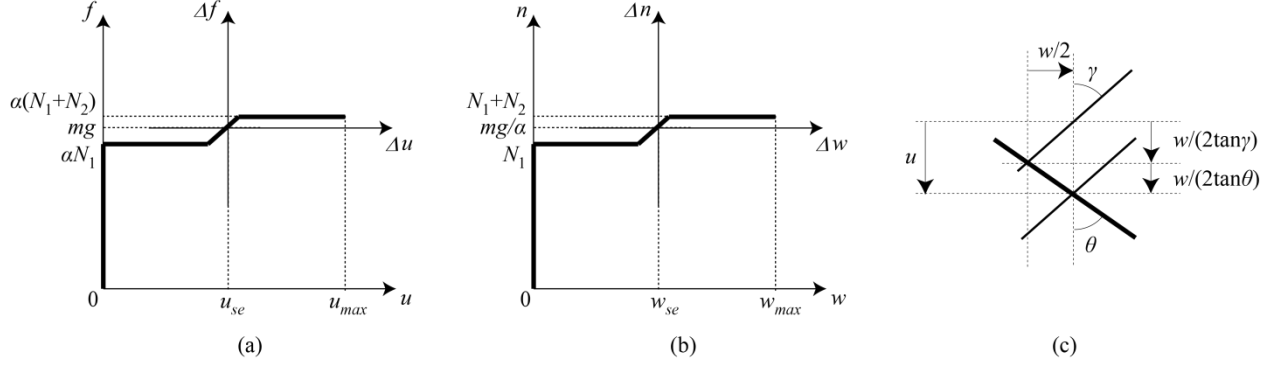


Figure 4: Mechanism for producing nonlinear restoring force. (a) Upper limit position. (b) Static equilibrium position. (c) Lower limit position.

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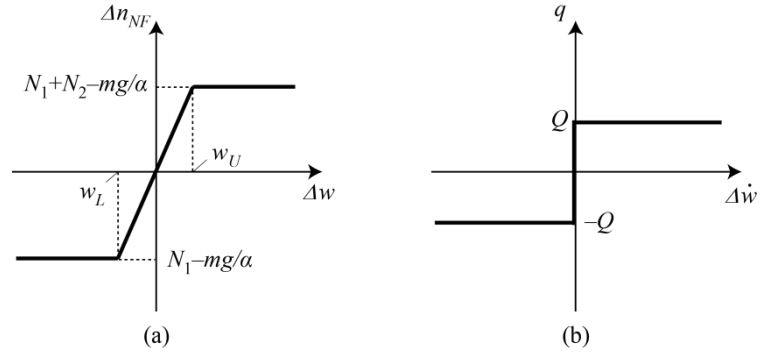
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Figure 5: Coordinates. (a) u - f and Δu - Δf . (b) w - n and Δw - Δn . (c) Geometric relationship between u and w .

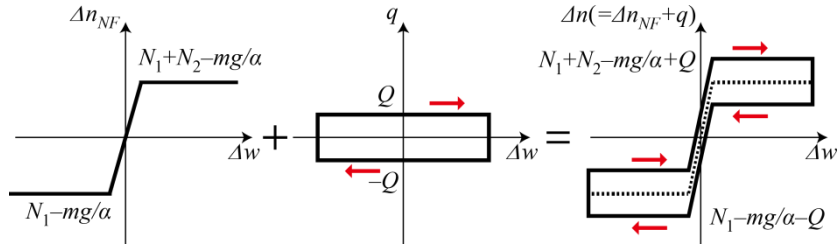


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Figure 6: Numerical models. (a) Restoring force. (b) Friction force.

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Figure 7: Schematic illustration of the restoring force model including friction.

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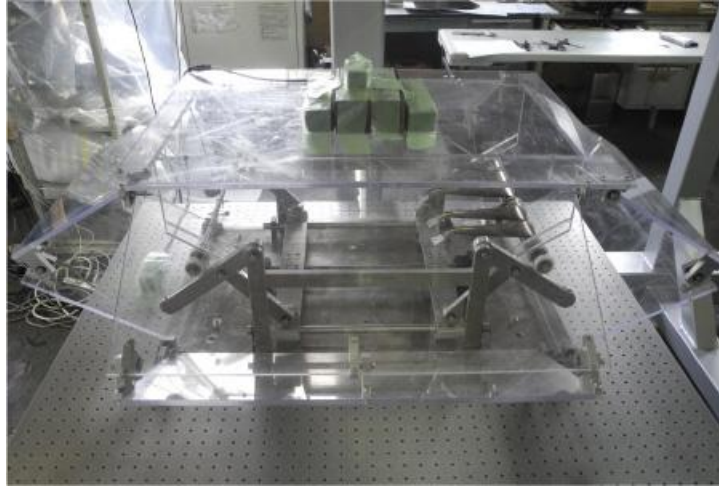


Figure 8: Photograph of a prototype of the vibration isolator.

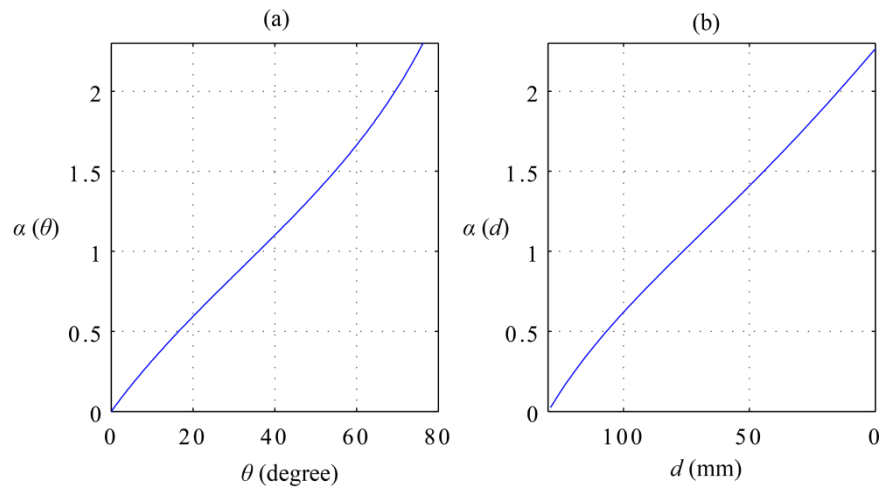


Figure 9: Amplification factor α . (a) α of θ . (b) α of d .

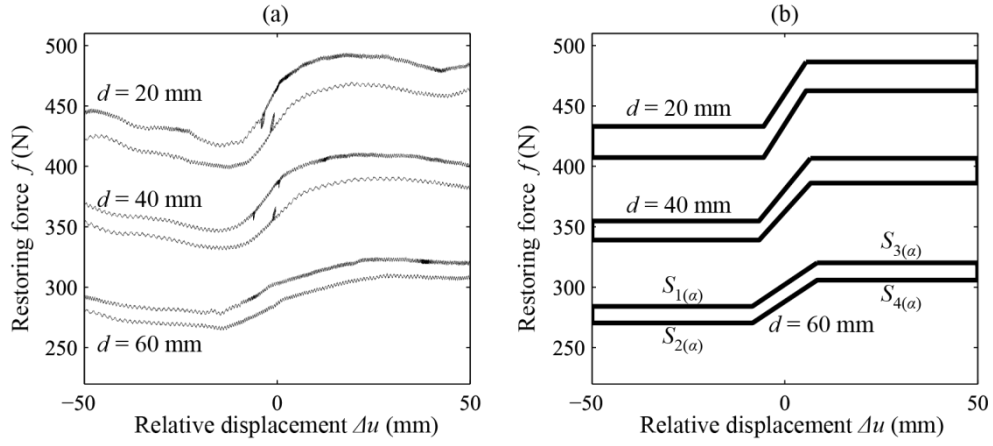


Figure 10: Restoring forces for the cases of $d = 60$ mm, $d = 40$ mm, and $d = 20$ mm. (a) Quasi-static loading test results. (b) Numerical models.

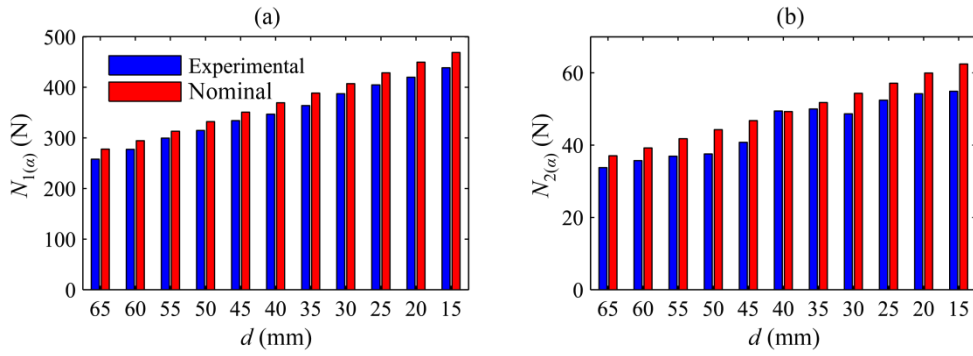


Figure 11: Comparisons between the experimental and nominal capacities of the constant-force springs. (a) $N_{1(a)}$. (b) $N_{2(a)}$.

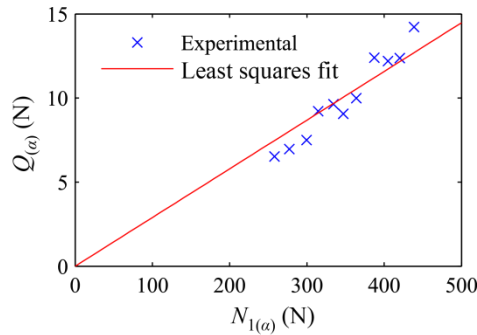


Figure 12: Relationship between $N_{1(a)}$ and $Q_{(a)}$.

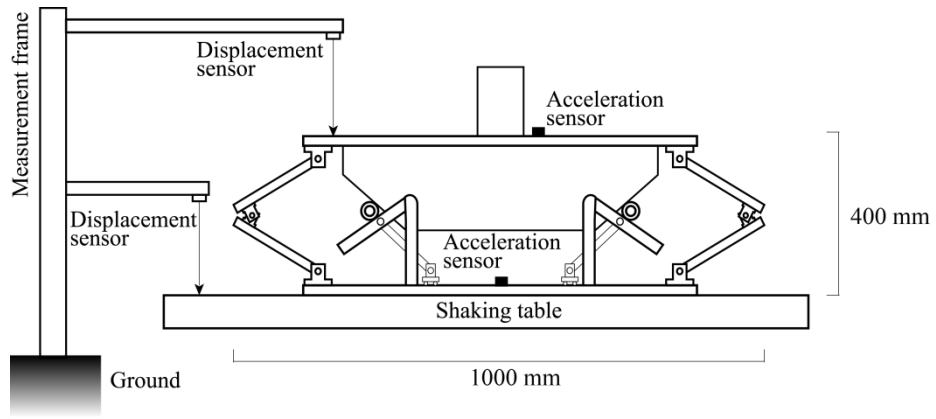


Figure 13: Experimental setup for the shaking table tests.

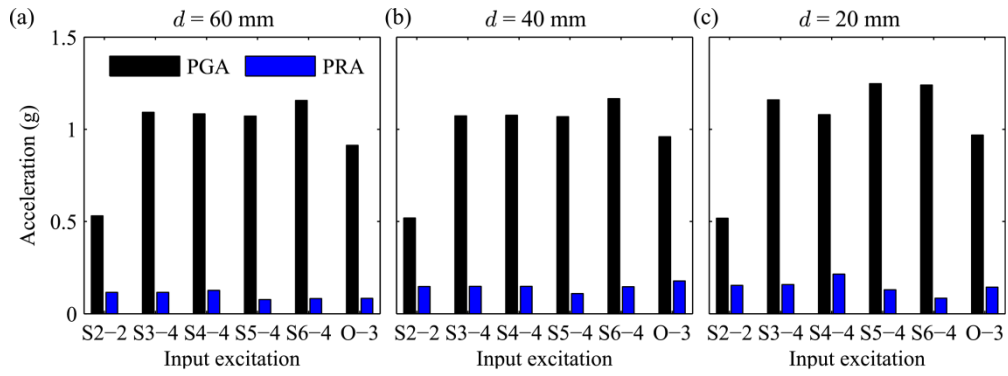


Figure 14: Comparisons of peak accelerations measured in shaking table tests. (a) $d = 60$ mm. (b) $d = 40$ mm. (c) $d = 20$ mm.

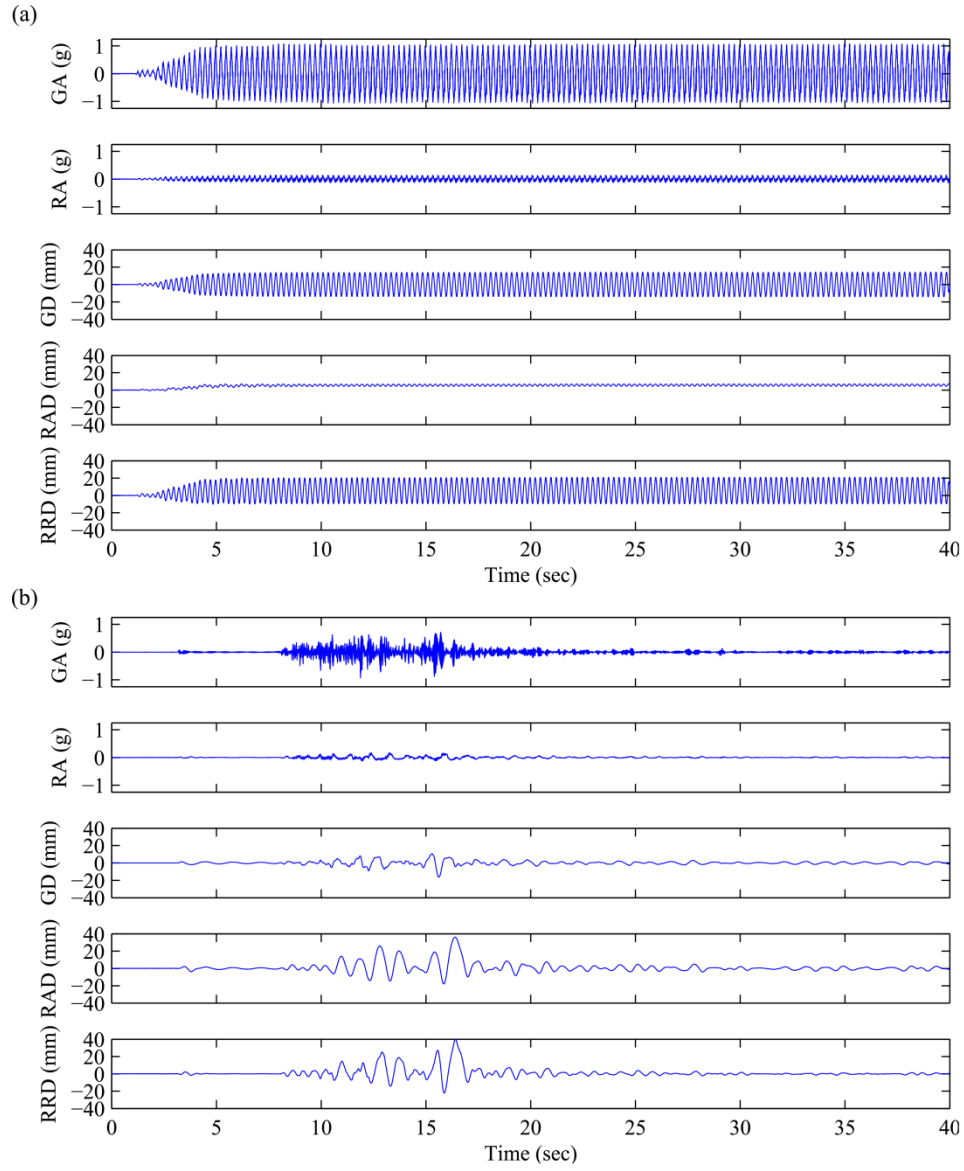


Figure 15: Time histories of shaking table tests for the case of $d = 40$ mm. (a) S4-4. (b) O-3.

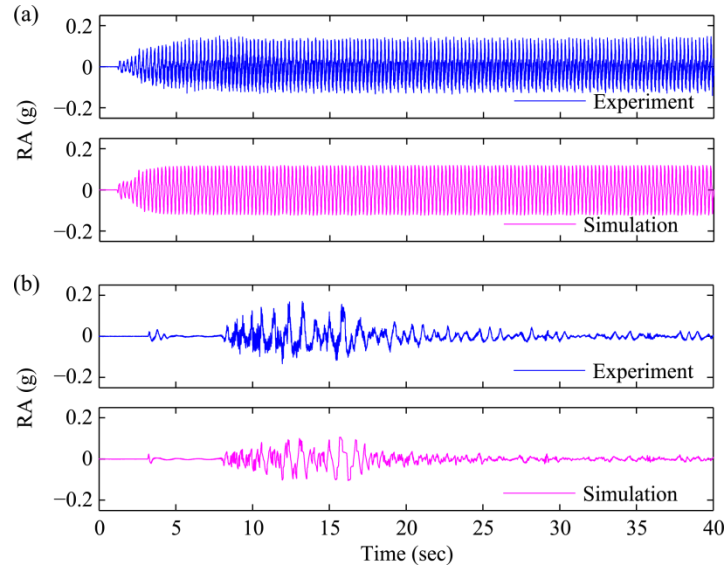


Figure 16: Comparisons of time histories between shaking table tests and simulations for the case of $d = 40$ mm. (a) S4-4. (b) O-3.

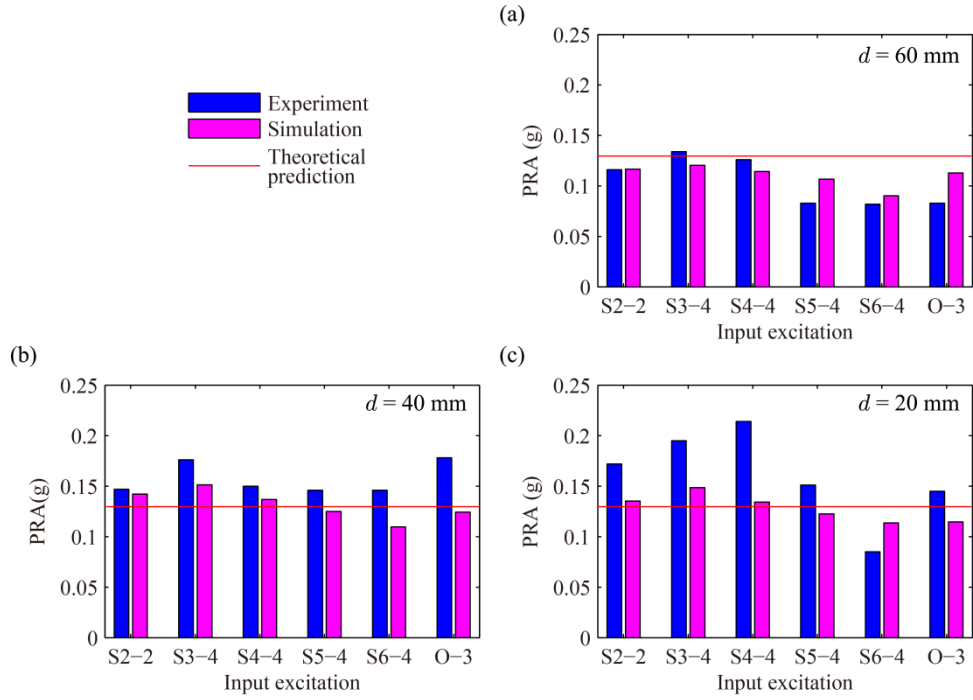


Figure 17: Comparisons of peak response accelerations. (a) $d = 60$ mm. (b) $d = 40$ mm. (c) $d = 20$ mm.

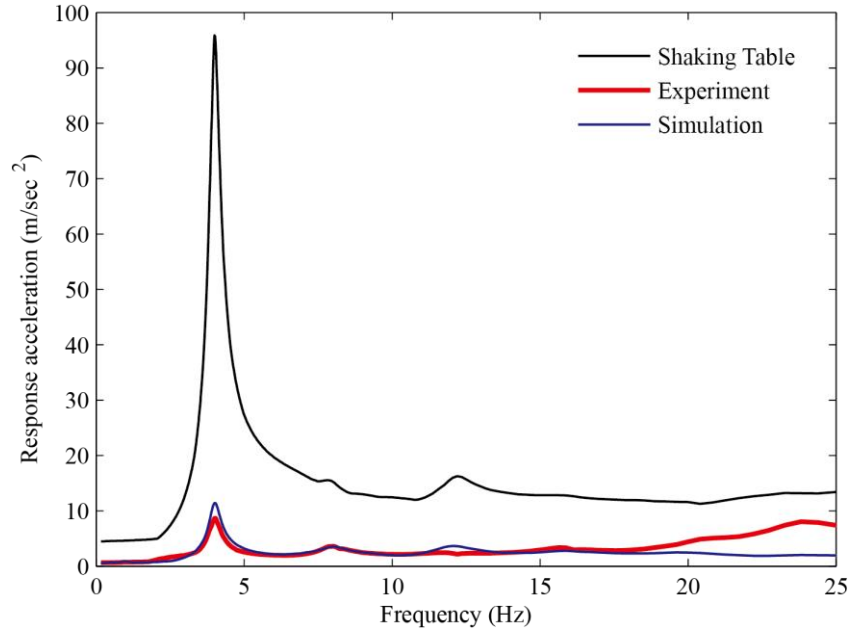


Figure 18: Response acceleration spectra for the case of S4-4 and $d = 20$ mm.

Table 1: Reference values of parameters.

d (mm)	α	$N_{1(\alpha)}$ (N)	$N_{2(\alpha)}$ (N)	$Q_{(\alpha)}$ (N)	$u_{R(\alpha)}$ (mm)	$k_{(\alpha)}$ (N/mm)
65	1.18	277.9	37.1	8.0	17.7	2.09
60	1.25	294.4	39.3	8.5	16.7	2.34
55	1.33	313.2	41.8	9.1	15.7	2.65
50	1.41	332.1	44.3	9.6	14.8	2.98
45	1.49	350.9	46.8	10.1	14.0	3.33
40	1.57	369.7	49.3	10.7	13.3	3.70
35	1.65	388.6	51.8	11.2	12.7	4.08
30	1.73	407.4	54.3	11.8	12.1	4.49
25	1.82	428.6	57.1	12.4	11.5	4.97
20	1.91	468.6	60.0	13.0	11.0	5.47
15	1.99	449.8	62.5	13.5	10.5	5.94

Table 2: Experimental values of parameters.

d	$S_{1(\alpha)}$ (N)	$S_{2(\alpha)}$ (N)	$S_{3(\alpha)}$ (N)	$S_{4(\alpha)}$ (N)	$N_{1(\alpha)}$ (N)	$N_{2(\alpha)}$ (N)	$Q_{(\alpha)}$ (N)	$k_{(\alpha)}$
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(mm)								(N/mm)
65	264.1	252.0	298.9	284.9	258.1	33.8	6.5	2.09
60	284.2	270.5	320.3	306.2	277.3	35.7	7.0	2.34
55	306.9	292.3	344.2	328.9	299.6	37.0	7.5	2.65
50	323.3	306.0	362.0	342.4	314.6	37.6	9.2	2.98
45	344.1	324.4	384.4	365.7	334.3	40.8	9.6	3.33
40	354.8	339.1	406.7	386.2	347.0	49.5	9.1	3.70
35	373.4	354.6	424.6	403.5	364.0	50.0	10.0	4.08
30	399.9	374.6	447.7	423.4	387.2	48.7	12.4	4.49
25	417.3	392.4	469.3	445.4	404.9	52.5	12.2	4.97
20	433.0	407.4	486.4	462.5	420.2	54.3	12.4	5.47
15	454.2	422.8	506.2	480.6	438.5	54.9	14.2	5.94

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Table 3: Input excitations.

Name	Input	Intended PGA (g)	Name	Input	Intended PGA (g)
S2-1	2 Hz sine wave	0.25	S5-1	5 Hz sine wave	0.25
S2-2	2 Hz sine wave	0.50	S5-2	5 Hz sine wave	0.50
S3-1	3 Hz sine wave	0.25	S5-3	5 Hz sine wave	0.75
S3-2	3 Hz sine wave	0.50	S5-4	5 Hz sine wave	1.00
S3-3	3 Hz sine wave	0.75	S6-1	6 Hz sine wave	0.25
S3-4	3 Hz sine wave	1.00	S6-2	6 Hz sine wave	0.50
S4-1	4 Hz sine wave	0.25	S6-3	6 Hz sine wave	0.75
S4-2	4 Hz sine wave	0.50	S6-4	6 Hz sine wave	1.00
S4-3	4 Hz sine wave	0.75	O-1	K-NET Ojiya UD	0.50
S4-4	4 Hz sine wave	1.00	O-2	K-NET Ojiya UD	0.75
			O-3	K-NET Ojiya UD	1.00

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557 Table 4: Comparisons between peak ground accelerations and peak response accelerations (g).

d	60 mm		40 mm		20 mm	
Excitation	PGA	PRA	PGA	PRA	PGA	PRA
S2-1	0.261	0.073	0.274	0.130	0.257	0.122

S2-2	0.531	0.116	0.519	0.147	0.518	0.154
S3-1	0.261	0.054	0.300	0.082	0.271	0.108
S3-2	0.522	0.086	0.525	0.122	0.631	0.129
S3-3	0.809	0.111	0.800	0.134	0.835	0.144
S3-4	1.092	0.116	1.073	0.148	1.159	0.158
S4-1	0.267	0.040	0.264	0.079	0.262	0.075
S4-2	0.537	0.063	0.565	0.077	0.525	0.094
S4-3	0.808	0.088	0.830	0.116	0.817	0.160
S4-4	1.083	0.126	1.076	0.148	1.079	0.214
S5-1	0.273	0.034	0.271	0.044	0.270	0.056
S5-2	0.535	0.046	0.529	0.068	0.532	0.087
S5-3	0.826	0.061	0.804	0.090	0.812	0.116
S5-4	1.071	0.077	1.068	0.109	1.248	0.130
S6-1	0.299	0.034	0.262	0.061	0.260	0.055
S6-2	0.548	0.045	0.614	0.087	0.547	0.070
S6-3	0.874	0.067	0.852	0.133	0.870	0.079
S6-4	1.156	0.082	1.166	0.146	1.240	0.084
O-1	0.429	0.061	0.473	0.146	0.494	0.132
O-2	0.680	0.072	0.706	0.160	0.698	0.133
O-3	0.913	0.083	0.960	0.178	0.968	0.144

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559 Table 5: Comparisons between peak ground displacements and peak response absolute
560 displacements (mm).

d	60 mm		40 mm		20 mm	
Excitation	PGD	PRAD	PGD	PRAD	PGD	PRAD
S2-1	14.089	6.084	13.691	8.919	13.608	9.647
S2-2	28.213	11.744	27.947	8.106	27.831	9.931
S3-1	6.366	1.622	6.167	2.994	6.092	3.094
S3-2	12.566	2.431	12.552	5.291	12.605	5.341
S3-3	18.883	2.569	18.700	6.697	18.661	6.981
S3-4	25.022	6.841	24.717	6.722	24.503	6.831
S4-1	3.650	1.428	3.573	1.175	3.595	1.209
S4-2	7.250	0.894	7.223	2.784	7.209	2.509
S4-3	10.923	1.219	10.636	4.550	10.863	3.806

S4-4	14.552	1.922	14.248	7.138	14.475	4.650
S5-1	2.341	0.913	2.327	0.613	2.338	0.691
S5-2	4.711	1.116	4.763	1.034	4.694	1.206
S5-3	7.141	1.319	7.081	1.641	7.128	2.066
S5-4	9.358	1.456	9.313	2.384	11.006	2.347
S6-1	1.677	0.344	1.641	0.406	1.663	0.378
S6-2	3.325	0.453	3.320	0.528	3.319	0.491
S6-3	5.009	0.469	5.000	0.978	4.994	1.066
S6-4	6.586	0.563	7.027	1.228	6.630	1.550
O-1	8.163	12.378	8.345	21.709	12.084	28.863
O-2	12.175	13.300	12.267	32.478	12.170	28.922
O-3	16.297	17.122	16.322	35.975	16.211	30.013

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